Trigonometry
...a synopsis.
by Dick Furnas

This picture is great! It summarizes practically all of the basic relations between the trigonometric functions at once.

Look closely. Since this is a unit circle (radius = 1), all the labels work by similar triangles and the following definitions:

- \( \sin \theta = \text{opp./hyp.} \)
- \( \cos \theta = \text{adj./hyp.} \)
- \( \tan \theta = \frac{\text{opp.}}{\text{adj.}} \)
- \( \cot \theta = \frac{\text{adj.}}{\text{opp.}} \)
- \( \csc \theta = \frac{1}{\sin \theta} = \text{hyp./opp.} \)
- \( \sec \theta = \frac{1}{\cos \theta} = \text{hyp./adj.} \)

- \( \tan \theta \) and \( \cot \theta \) even lie in a line tangent to the circle.

- Every right triangle in the picture gives you a trig identity by the Pythagorean theorem:
  
  \[
  \sin^2 \theta + \cos^2 \theta = 1 \\
  1 + \tan^2 \theta = \sec^2 \theta \\
  1 + \cot^2 \theta = \csc^2 \theta 
  \]

  to the ridiculous (try this out on your friends in Engineering!)

  \[
  \csc^2 \theta + \sec^2 \theta = (\tan \theta + \cot \theta)^2 
  \]

Points to ponder:

- How long is the arc from the x-axis to where the radius intersects the circle?
- How long would it be if the radius were 2?
- How would the labels in the picture have to change if the radius were 2? (the unit circle is nice, eh!)
- This picture shows \( \theta \) in the first quadrant. How would it be different if \( \theta \) were in each of the other quadrants?
- In particular, in each quadrant what happens to the sign (plus or minus) of:

<table>
<thead>
<tr>
<th>Quadrant</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin \theta )</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \csc \theta )</td>
<td>+</td>
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<tr>
<td>( \cos \theta )</td>
<td>+</td>
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<tr>
<td>( \sec \theta )</td>
<td>+</td>
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<tr>
<td>( \tan \theta )</td>
<td>+</td>
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</tr>
<tr>
<td>( \cot \theta )</td>
<td>+</td>
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</tbody>
</table>

- If the x- and y- axes are labelled as usual, even most of the signs above "take care of themselves" from the picture. (i.e. The coordinate axes show you the sign directly.)

- Which do not?

\[
\begin{array}{|c|c|c|c|}
\hline
& I & II & IV \\
\hline
\sin \theta & + & & \\
\csc \theta & + & & \\
\cos \theta & + & & \\
\sec \theta & + & & \\
\tan \theta & + & & \\
\cot \theta & + & & \\
\hline
\end{array}
\]