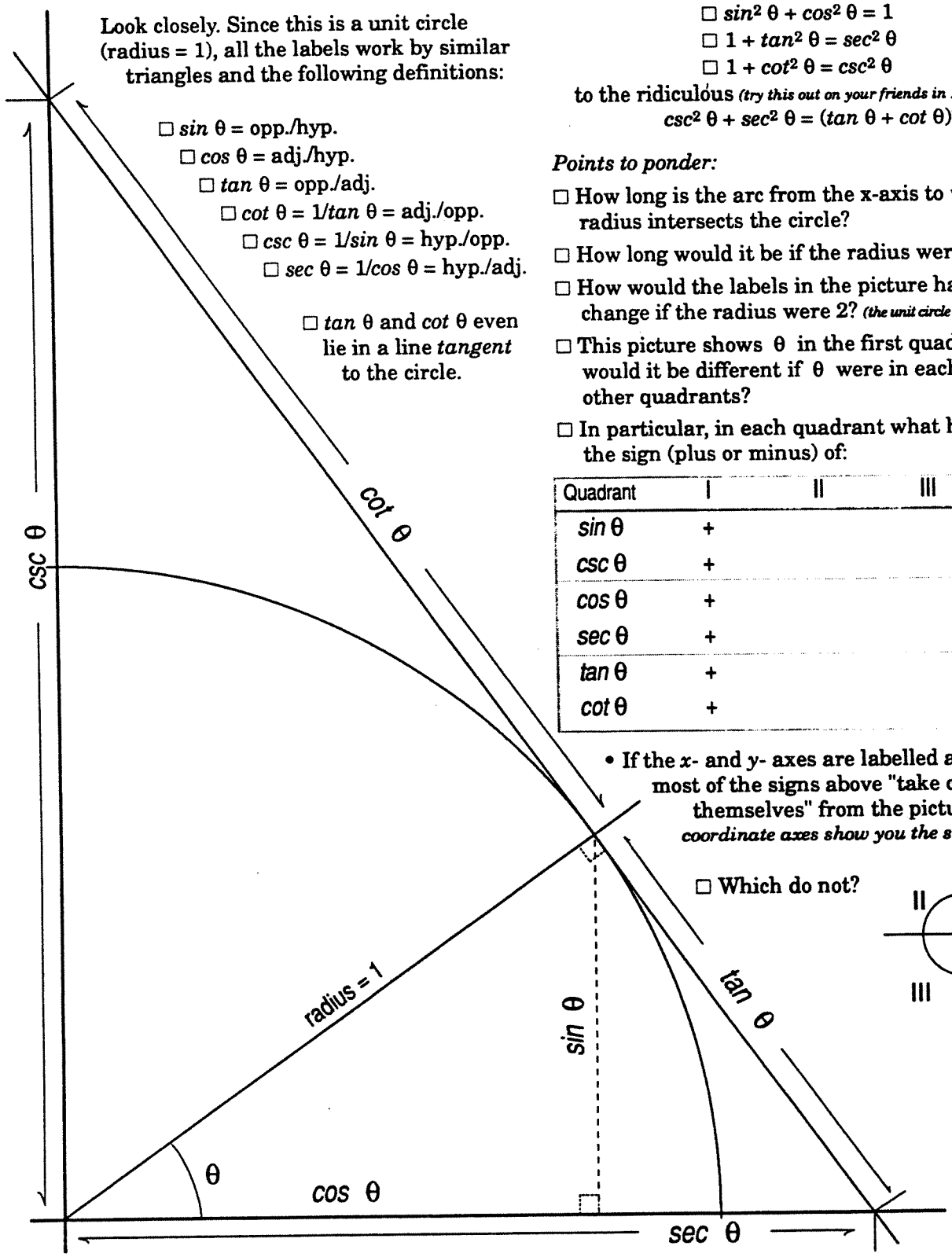
Trigonometry

...a synopsis.

by Dick Furnas

This picture is great! It summarizes practically all of the basic relations between the trigonometric functions at once.



Look closely. Since this is a unit circle (radius = 1), all the labels work by similar triangles and the following definitions:

- ☐ *sin* θ = opp./hyp.
- ☐ *cos* θ = adj./hyp.
- ☐ *tan* θ = opp./adj.
- ☐ *cot* θ = 1/*tan* θ = adj./opp.
- ☐ *csc* θ = 1/*sin* θ = hyp./opp.
- ☐ *sec* θ = 1/*cos* θ = hyp./adj.

☐ *tan* θ and *cot* θ even lie in a line *tangent* to the circle.

• Every right triangle in the picture gives you a trig identity by the Pythagorean theorem:

from the sublime *(and famous)*

- ☐ $\sin^2\theta + \cos^2\theta = 1$
- ☐ $1 + \tan^2\theta = \sec^2\theta$
- ☐ $1 + \cot^2\theta = \csc^2\theta$

to the ridiculous *(try this out on your friends in Engineering!)*

$$\csc^2\theta + \sec^2\theta = (\tan\theta + \cot\theta)^2$$

Points to ponder:

- ☐ How long is the arc from the x-axis to where the radius intersects the circle?
- ☐ How long would it be if the radius were 2?
- ☐ How would the labels in the picture have to change if the radius were 2? *(the unit circle is nice, eh?)*
- ☐ This picture shows θ in the first quadrant. How would it be different if θ were in each of the other quadrants?
- ☐ In particular, in each quadrant what happens to the sign (plus or minus) of:

Quadrant	I	II	III	IV
sin θ	+			
csc θ	+			
cos θ	+			
sec θ	+			
tan θ	+			
cot θ	+			

• If the *x*- and *y*- axes are labelled as usual, even most of the signs above "take care of themselves" from the picture. *(i.e. The coordinate axes show you the sign directly.)*

☐ Which do not?